The euler–maruyama approximation for the asset price in the mean-reverting-theta stochastic volatility model

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Abstract

The trading of financial derivatives and products in financial markets has influenced the development of the world economy. Over the last few decades, a rapid growth in complex financial systems, which can generate unstable conditions in financial markets, has been observed. Therefore models are being developed to study and examine the uncertainty surrounding these financial systems in different circumstances. The important milestone of this work can be traced to the Black-Scholes formula for option pricing which was published in 1973 and revolutionized the financial industry by introducing the no-arbitrage principle. This model assumed that the average rates of return and volatility are constant however, this is not realistic. Therefore, several models have been developed, based on pragmatic studies, which generalize the Black-Scholes formula to acquire more knowledge for these financial systems.

In this project, we focused on the following mean-reverting-theta stochastic volatility model in finance which did not have explicit solutions.

$$dX(t) = \alpha_1(\mu_1 - X(t))dt + \sigma_1\sqrt{V(t)}X(t)^\theta dw_1(t)$$
$$dV(t) = \alpha_2(\mu_2 - V(t))dt + \sigma_2 V(t)^\beta dw_2(t)$$

where α_i, β_i and σ_i for $i = 1,2,3$ are constant and $\theta, \beta > 1.$ We first developed a technique to prove the non-negativity of solutions to the model. We then showed that the Euler–Maruyama (EM) numerical solutions will converge to the true solution in probability. We also showed that the EM solutions can be used to compute some financial quantities related to the Stochastic Differential Equations (SDEs) models including the option value, for example.

Keywords: Euler–Maruyama method, Stochastic differential equation, Brownian motion, Option value