Warehouse Space Optimization Using a Linear Programming Model

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***Abstract*— Warehouse operations play a key role in manufacturing organizations, providing support to run production processes smoothly. As a part of warehouse operations in a supply chain, loading problems are a point that allows optimization to be carried out with significant cost implications and a considerable impact on the rest of the supply chain in manufacturing organizations. The paper proposes a simple and effective linear programming model to optimally use warehouse storage space by efficient palletizing. The quantity of total pallets required per day is derived based on the available demand per day and other constraints related to warehousing operations in a multi-product manufacturing context. Requiring moderate computational efforts and time, generates optimal solutions of almost all examples analyzed, using the Microsoft Excel add-in, Solver and the Solver Engine - Simplex Linear Programming, which were used to implement the model. For scenarios considered, and a sample scenario demonstrated in the paper, the model generated feasible solutions, all constraints are satisfied.**

Keywords—palletization problem, warehouse optimization, linear programming.

I. INTRODUCTION

Material handling and inventory controlling are critical functions of supply chain management [18]. When supply chain processes are evaluated, loading problems are considered a prime problem faced by modern supply chain managers [1].

When supply chain processes such as logistics and distribution management are evaluated from a cost standpoint, various direct costs such as costs incurred with physical handling of goods, transportation of goods from one point to another, storage of finished goods, raw materials, work in progress inventories and administration cost, and indirect costs such as capacity cost and shortage cost could be identified [19].

When costs and savings related to processes of the supply chain are compared, a direct saving on total logistics cost could be achieved through increased efficiency of pallet loading, irrespective of the type of transport. These savings range between 20-25% of the storage cost and up to 30% of transportation costs [3], [5], [13] and [23].

Therefore, optimization of the palletizing function yields practical and empirical benefits, not only in the transportation industry, but also in supply chain management, due to upstream and downstream influence in the logistics flow [4], [11]. (Palletizing refers to a place, stack, or transport (goods) on a pallet or pallets where a "pallet" is a flat transport structure that supports goods in a stable fashion while being lifted by a forklift, pallet jack, front loader, work saver or other jacking device. A pallet is the structural foundation of a unit load, which allows handling and storage efficiencies.

II. LITERATURE REVIEW

A. Warehousing and supply chain operations

Warehouses play a key role manufacturing organization providing support to run in production processes smoothly. Warehouse operations include tasks such as receiving and unloading raw materials, optimum location of storage goods in the warehouse according to the dispatch orders etc. [18]. Optimal management of warehouse operations is critical for manufacturing organizations as warehouses are a central entity that directly and indirectly interact with several tasks of production processes, impacting the supply chain of the manufacturing industry [17]. If managed properly, warehouse operations could speed up production processes and reduce cost incurred due to process delays [25].

The main objective of maintaining a warehouse is shifting finishing goods, raw materials and work in progress inventories from any configuration to the next stage of the supply chain effectively.

In most operational environments, warehouse efficiency requirements are not considered in advance. Consequently, it will lead to complications in logistics activities in organizations of the manufacturing industry, since warehouses act as a hub of supply chain which coordinate and glue all activities and tasks [10]. Therefore, warehouse operations optimization has become more complicated in the modern business era, with the emergence of globalized supply chain processes and its contradiction with traditional management and optimization practices. In such scenarios, taking steps to execute optimization techniques to store finish goods, raw materials and work in progress inventories in order to decide when and how much to deliver from every origin to its relevant destination within a specific time period is a proper way to make warehouse process improvements [14].

B. Optimization Approaches

Linear optimization is a method for the optimization of an objective function that is subjected to several equalities and inequalities of constrains [14] and is used to obtain the lowest cost or the highest profit in a developed mathematical model, with identified requirements, assuming linear relationship between its variables [9].

The Pallet Loading Problem (PLP) arises when small items are placed onto a large pallet in manufacturing workshops and other logistic areas; specifically, cargo, with similar or different dimensions and weights must be packed into a rectangular pallet, efficiently, to increase pallet capacity utilization and stability by assuming dimensions and weights are known [16]. The objects being stacked may be of different sizes, nature, weight, value and shapes. Therefore, when finding solutions for pallet loading problems, the above mentioned factors and several measurement criteria such as pallet capacity utilization, pallet stability, pallet strength, humidity and time of storage should be considered [16].

The most important constraints related to the PLP are order demand, placement and weight conditions and stack stability requirements while other constraints such as limitations on stack height, profit, preservation of the objects are also identified [8].

PLP falls in 2 categories,

- The Manufacturer's Problem: when the objects to be stacked on to a pallet are homogenous.
- The Distributor's Problem when the objects to be stacked on to a pallet are heterogeneous [6], [16].

The PLP has been reviewed by a number of scholars' time to time, resulting in the introduction of several solutions [21].

[3] have suggested a discrete event system approach to decrease logistics and distribution cost of an organization.

[16] have extended the problem by adding strength of loading bearing, vertical and horizontal stability as constraints. At the end they have presented a 0-1 ILP model as a solution.

[2] have suggested a MILP model by considering the load balancing problem, concluding that the model is not practically applicable to small instances where less than 25 cargo items exist.

Many scholars have proposed solutions to the PLP using heuristics and metaheuristics.

[6] have mainly reviewed manufacturers PLP (MPLP) and reduced the three-dimensional MPLP up to two dimensional MPLP. Additionally, they have compared the heuristic solution with exact optimization outcomes, considering similar benchmark instances.

It is also observed that most studies have treated the PLP as bi-dimensional, excluding height, to simplify it [22].

[12] have introduced a heuristic approach based on simulated annealing, considering expiration of order dates. The proposed algorithm mainly aims to maximize usage of container space while considering time expiration of orders.

[24] have reviewed the open dimensional packing problem in global optimization and presented a methodology to reduce computation time by reducing the number of binary variables and linear equality constrains.

It could be observed that a majority of studies on PLP (a summarized comparison of the different approaches to the PLP is presented in [22]) use exact algorithms with the incorporation of heuristics. The involvement of heuristics in the PLP could be observed in cases of increased computational complexity and time affect the problem, and as the problem is identified to be NP hard as determined by the constraints and scope modelled. Simulation approaches are also used where the parameters display stochasticity.

Solutions that have modelled the problem in a complex form are found to consume a significant amount of computational resources in addition to taking a long time to generate results. Considering available reviews on literature it is observed that the existing approaches for the PLP only model a few of the considerations present in reality. Stability of stacks has been the most widely considered factor in the models, considering the requirements of transportation and storage [22]. The approaches that employ heuristics such as the wall building, taboo search, simulated annealing etc. are found to generate non-efficient/non-optimal final solutions, as an inherent tradeoff of heuristic approaches where the optimality of the output is compensated for, by the reduced time consumption and computational complexity involved. According to literature, the most important advance in solutions lies in the fact that the models are flexible enough to accept different information flows to tackle the PLP according to the operational context to be optimized [15].

According to the involvement of optimization techniques and the scope of the problem considered which is to optimize the use of warehouse storage space using the palletization function, in a situation where the demand is previously known by the warehouse managers in the form of orders, the approach proposed in the study uses linear programming as the optimization methodology. This pertains to the models and solutions proposed in available literature in the optimization technique used. Most studies have proposed well performing solutions to the problem using heuristics and advanced optimization techniques. It is the purpose of this study to propose a simple yet practically effective solution to warehouse operations where the scale of the problem, therefore, the solution space is not exponentially large. In addition, since the proposed model provides exact optimization, it is expected to provide practical managerial assistance and utilize minimum computation and complexity of computations in addition to solution time and work on simple computational software. With respect to the gaps identified in the literature survey, and the industrial context considered, the methodology suggested in this study is a linear programming model.

III. Methodology

The model proposed is concerned with the optimal use of warehouse storage area with relevance to the palletization function in its supply chain. The quantity of total pallets required per day is derived based on the available demand per day, so as to best utilize warehouse space. Using the model proposed, the number of pallets required to be stored in the warehouse could be found, that allows optimal usage of warehouse storage area and satisfies all the demand in a multi-product manufacturing context.

Decision variable: X_{ik} = the number of kth pallet type from i^{th} product type (for i=1,2,….n & k=1,2,….,r)

Objective function: The maximum number of pallets needed to fulfill the demand from each product type.

Max Z = $C_k \sum_{i=0}^{n} X_{ik}$ (for k=1, 2, …., r)

Constraints -

The produced total number of pallets from each product type must be equal to or less than the daily production availability of each product type.
$\sum_{i=1}^{n} C_k X_{ik} \leq P_i$ (for k=1, 2, …., r)

The total number of produced pallets from each product type must be equal to or less than the daily pallets requirement from each product type.
$\sum_{i=1}^{n} X_{ik} = \sum_{i=1}^{n} A_i Y_{ij}$ (for k=1, 2, ……, r & j=1, 2, …., m)

The number of pallets according to the placed order from each product type must be equal to or less than the capacity of warehouse.
$C_k \sum_{i=1}^{n} X_{ik} \leq M$ (for k= 1, 2, …., r)

The number of pallets is integer and greater than zero.
$X_{ik} \geq 0$ (for i=1, 2, …, n & k=1, 2, …., r)

Main Parameters:
n = total number of brand types (product type)
m = total number of order types
r = total number of pallet groups
C_k = the quantity type of pallet in group k (for k = 1, 2, …, r)
P_i = the available capacity of product type i (for i=1, 2, …., n)
Y_{ij} = the number of j^{th} order type from i^{th} product type according to daily requirement (for i=1,2 … ,n & j=1,2,….,m)
M = the capacity of warehouse

Consider the sample situation represented in table.2, where the number of pallets that can fulfill a certain amount of demand is shown. Assume that each type of pallet requires an equal storage area and can only be stacked in ground level (no stack is placed over another stack). According to the table below (Table 2:), the highest number of pallets (22 pallets) is required for a demand for a 1000 units and other three types of demand quantities require an equal number of pallets (12 pallets).

Table 1: Number of Pallets for certain order

Demand	Pallet Types			
	50	45	35	25(Manual)
1000	2	20	-	-
600	12		-	-
580	4	8	-	-
400	-	-	10	2

For a sample case taken from the cement manufacturing industry, where maximum capacity of warehouse is 30,000 units and three product types (AA, AB, AC) are available with 10000 units' capacity from each product type, and daily demand requirements are as below (Table 2).

Table 2: Daily Demand Requirement

Product	Order Type			
	1000	600	560	400
AA	5	7	3	1
AB	8	5	4	2
AC	3	3	1	3

Max Z = $50 \sum_{i=1}^{3} X_{i1} + 45 \sum_{i=1}^{3} X_{i2} + 35 \sum_{i=1}^{3} X_{i3} + (0.2*25\sum_{i=1}^{3} X_{i3})$

Constraints
$50X_{11} + 45X_{12} + 35X_{13} + 0.2*25X_{13} <= 10,000$ → (for product type AA)

$50X_{21} + 45X_{22} + 35X_{23} + 0.2*25X_{23} <= 10,000$ → (for product type AB)

$50X_{31} + 45X_{32} + 35X_{33} + 0.2*25X_{33} <= 10,000$ → (for product type AC)

$X_{11} + X_{21} + X_{31} <= 2(Y_{11} + Y_{21} + Y_{31}) + 12(Y_{12} + Y_{22} + Y_{32}) + 4(Y_{13} + Y_{23} + Y_{33})$ → (for pallet type 1)

$X_{12} + X_{22} + X_{32} <= 8(Y_{11} + Y_{21} + Y_{31}) + 20(Y_{13} + Y_{23} + Y_{33})$ → (for pallet type 2)

$X_{13} + X_{23} + X_{33} <= 10(Y_{41} + Y_{42} + Y_{43})$ → (for pallet type 3)

Lower bound for decision variable

$X_{i1} >= 12$ → (if 600 units order placed lower bound for pallet type 1 where i=1, 2, 3)
$X_{i1} >= 4$ → (if 560 units order placed lower bound for pallet type 1 where i=1, 2, 3)

$X_{i1} >= 2$ → (if 1000 units order placed lower bound for pallet type 1 where i=1, 2, 3)

$X_{i2} >= 20$ → (if 1000 units order placed lower bound for pallet type 2 where i=1, 2, 3)

$X_{i2} >= 8$ → (if 560 units order placed lower bound for pallet type 2 where i=1, 2, 3)

$X_{i3} >= 10$ → (if 400 units order placed lower bound for pallet type 3 where i=1, 2, 3)

X_{ik} → integer

$Y_{ij} >= 0$ (for i=1,2,3 & j=1,2,3,4)

$P_i >= 0$ (for i=1, 2, 3)

IV. Results and discussion

For the sample scenario considered, the solution is feasible and is an integer solution within tolerance, with a maximum value for objective function (Max Z) as 29,965 units and optimal decision variables values shown in table 3. All constraints are satisfied. Optimal decision variables values shown in table 3. All Constraints are satisfied.

Table 3: Optimal Results for Decision Variables

X_{ik}	Pallet Type(k)		
Product(i)	1(50)	2(45)	3(35)
1(AA)	12	200	10
2(AB)	68	137	10
3(AC)	174	20	10

According to the optimal values of decision variables it is found that, manufacture from each product type (product) AA = 10,000 units, AB = 9,965 and AC = 10,000 units should be produced.

The software used is the Microsoft Excel add-in, Solver, using the Solver Engine - Simplex LP, where the Solution Time was 0.047 Seconds.

V. Managerial implications

Transportation activities and tracking motions increase the logistics cost of organizations. Cost incurred due to transportation activities and trucking motion are mainly based on two things, volume of shipment and utilization of organizational transportation capacity. Therefore, organizations in manufacturing industry should pay higher attention to optimizing the number of units packed into pallets, because it leads to increased efficiency in transportation and tracking activities. The palletizing process is an important link in all the logistics functions where orders from customers must be fulfilled. Considering cost competition, conducting palletization in an optimal way for the issue is critical. Recent issues in transportation and storage costs have highlighted the importance of an optimal palletization, both in transit and in the warehouse.

The proposed model calculates the number of pallets required to be stored in the warehouse. Which consequently allows optimal usage of warehouse storage space. Warehouse costs are directly related to the type and number of pallets used for a shipment, which, in turn, is directly linked to proper space utilization. At the same time, achieving an optimal number of pallets reduces the shipment while increasing the stability and support of the load.

Time spent on the palletization process and the space utilization which is defined to be the percentage of pallet space or volume occupied by the load products are optimized. Because arrangement of cargo into a pallet is time-consuming, and requires stability and space considerations with regards to the utilization of warehouse space. It will lead to reduce the total loading cycle time of the supply chain. Through the findings of the proposed model, a valuable insight into how logistics managers would reduce container loading time of particular organization indirectly while maximizing warehouse storage, is provided.

As outcomes achieved through the proposed model, indirect minimization of the unnecessary labor hours and labor motions could also be observed. Logistics and distribution mangers do not directly load goods from manufacturing process to containers. Instead, at the end of the production process, all finished goods are directly sent to the central warehouse of the supply chain. They directly load their order only from warehouse to containers. Hence, the operational context of the palletization process has a greater probability to be relatively simple, and not become NP hard. Whether logistics and distribution mangers take steps to directly load goods from manufacturing process to containers or not, they should definitely send goods to the central warehouse. Therefore, it is observed that, the proposed model will lead to reduced routines in transportation systems by a significant amount as well.

VI. Conclusion

This paper presents a simple and effective linear programming model to optimally use warehouse storage space by efficient palletizing. The quantity of total pallets required per day is derived based on the available demand per day and other constraints related to warehousing operations in a multi-product manufacturing context. Requiring moderate computational efforts and time, the model was able to generate optimal solutions of almost all examples analyzed. This suggests that the proposed LP is an effective solution to the problem of optimizing the number of pallets needed to fulfill the demand from each product type, maximizing warehouse space utilization. The approach proposed in this paper can be used as a basis for further optimization of basic warehousing functions in a practical environment where the solution space is not exponentially large, such as that of a single warehouse.

VII. Limitations and future research

The proposed model of this study is only focused on optimizing ground floor space utilization in a particular manufacturing organization, there is a need for future research to develop a model for optimizing not only ground floor but also storage space available for vertical stacking.

The proposed model of this study mainly takes daily demand as input. Using statistical methods (Ex. Forecasting) logistics and distribution managers could determine order requirements in advance. The research could be further extended to incorporate forecasted order quantity as inputs to the model. Furthermore, the proposed model of this study considers the fact that objects that are stacked on to a pallet are homogenous. The model could also be tested and improved to perform in a situation where the objects to be stacked on to the pallet are heterogeneous. This model could also be developed using goal programming when there is order priority, and are possible avenues for future research.

VIII. References

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