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WAREHOUSE SPACE OPTIMIZATION USING LINEAR PROGRAMMING MODEL AND GOAL PROGRAMMING MODEL

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Abstract

Warehousing involves all the material handling activities that take place within a warehouse in a supply chain. Typical warehouse operations involve integrated assignment decisions that could be optimized. The review in this paper presents a comparative analysis of optimization models used for warehouse space allocation proposed in the recent literature. Through the review, it was identified that considerable challenges exist in applying these models at present, due to the vast amount of information required to be processed, the significant number of possible alternatives, and the degree of integration of decisions required in the modern warehousing context. The objectives of this paper are comparison of previous research related to the warehouse space optimization and demonstrate warehouse space optimization using linear programming (LP) and goal programming (GP). Therefore, it is expected that the paper serves as a reference for further research in the area. The paper proposes a simple and effective linear programming (LP) model and goal programming (GP) model to optimize warehouse storage space by efficient palletizing. The quantity of total pallets required per day is derived based on the available demand per day and other constraints related to warehousing operations in a multi-product manufacturing context. The models generated feasible solutions; all constraints are satisfied.

Keywords: goal programming, linear programming, optimization models, warehouse operations, warehouse space utilization

INTRODUCTION

Warehousing is concerned with all the material handling activities that take place within a warehouse. Typical warehouse operations could be identified as assignment problems such as Assignment of items to different warehouse departments, Space allocation, Assignment of (stock-keeping unit) SKUs to zones, Assignment of pickers to zones, Storage location assignment, Specification of storage classes (for class-based storage), Batch size, Order-batch assignment (Manzini et al., 2011). Warehouse operations broadly include the receiving of products, storage, order-picking, accumulation, sorting, and shipping operations. Warehouses could be categorized into 3 sectors based on operations (Geraldes, 2003): distribution warehouses, production warehouses, and contract warehouses (Berg & Zijm, 1999). A distribution warehouse is a warehouse in which products from different suppliers are collected (and sometimes assembled) for delivery to several customers. A production warehouse is used for the storage of raw materials, semi-finished products, and finished products in a production facility. A contract warehouse is a facility that performs the warehousing operation on behalf of one or more customers. A detailed discussion on warehousing systems and a classification of warehouse management problems is presented in (Berg & Zijm, 1999).

Warehouse operations optimization can be divided into three segments according to its activities (M. T. Hompel & Schmidt, 2007). The first one is the basic technical structure of the warehouse, the second one is the operational and organizational framework, the third one and special part are the coordinating and controlling systems for warehouse operations.

By introducing Flying-V and Fishbone (Pohl et al., 2009a) design of cross aisles under new basic technical structure, warehouse operations can utilize up to 10%-20% of traveling distance. With the help of the introduction of new technologies such as Bar-Codes, RFID chips, Magnetic strips, or Machine vision (Karasek, 2013) warehouse managers can sort materials (raw materials, in-progress, and finished goods) and store them more efficiently while saving labor energy and optimizing the available space. To optimize the operational structure, what is heavily used, as was identified through reviewed literature, is family grouping where they arrange space-related or similar products or orders by closer or best locations from input/output gates (Roodbergen & Koster, 2010; Schuur, 2007). S-shape, return policy, Mid-point strategy, largest gap strategy, Composite heuristic, Optimal routing are some space arranging metho13ds for single block warehouses (Clarke & Wright, 1964) whereas order-batching, simply-batching are some space arranging methods in multi-block warehouses (Management, 2008).

Around 20% to 50% of the total operating expenses in manufacturing companies are attributed to goods storage and handling (Ozgormus & Smith, 2020). Warehouses frequently involve large expenses such as investments for land and facility equipment (storage and handling activities), costs connected to labor-intensive activities and information systems (Ballesteros-riveros et al., 2019). Efficient facility planning is found to be able to reduce around 10% to 30% of those costs (Manzini et al., 2011).

LITERATURE REVIEW

(Manzini et al., 2011) presents a model based on previous work, for allocating warehouse space to several products in a defined time horizon using GAMS © software and solved using the CPLEX 12.6 solver. The model aims to reduce the holding and material handling cost in a multiproduct environment. The model allows the simultaneous assignment of the products to the different areas (cross-docking, reserve, and shipping) and the dynamic assignation of the spaces in the warehouse in the several time horizons, in which the demands and the costs change. (Behnamian & Eghtedari, 2009) states that a basic rule in assigning products to storage locations is storing "better" products in the "better" locations in the order picking system. "The most desirable location" is a location that provides faster and more ergonomic access to the product stored. (Behnamian & Eghtedari, 2009)presents various optimization models that contain multiple products with stores and retrieves performed in a single command model (i.e., there is only one store or one retrieve on each round trip). Most of the models consider constraints or assumptions such as operability, compatibility of storage products, replenishment methods, routing, worker efficiency in order preparations and retrieval, and technology usage (Husseini, 2011; Operational, 2018). (Heragu et al., 2007) consider product allocation and the warehouse sizing problem through both a mathematical optimization model and a heuristic based on a branch and bound algorithm integrating product allocation and functional area size determination in a warehouse to minimize annual handling and storage costs. Dynamic programming models could be found for warehouse optimization problems where block-stacking occurs (storing products without using shelves). (Caserta et al., 2011) has proposed a dynamic programming model and metaheuristic solution algorithm for the blocks relocation problem that could be adopted for optimizing warehouse space allocations. Similar block relocation algorithms proposed in (Maglić et al., 2020) based on a branch and bound algorithm and a genetic algorithm heuristic find the best sequence of retrievals base on four heuristically defined rules to minimize the number of relocations. The study in (Manzini et al., 2011) presents a framework for warehouse design and operation problems, where a classification of warehouse design and operation planning problems is discussed in detail. In (Operational, 2018) a mathematical linear formulation is proposed for the Products Allocation Problem in a multi-layers warehouse with compatibility constraints among the classes. An Iterated Local Search (ILS) has been defined using CPLEX software to compensate for time when solution space is large. The objective of the model is to minimize the delivery time, inventories, total logistic costs. Further, it is experimentally shown that the proposed heuristic performs better, with less computational complexity than the mathematical model proposed in the paper.

(Geraldes, 2003) presents a mixed-integer nonlinear programming model (MINLP) that integrates the decisions of warehouse size, additional storage capacity available, replenishment quantities and re-order points in warehouse design and inventory operations. Ozgormus & Smith, (2020) suggests that using maximum on-hand quantities if the replenishment orders of all products arrive at the same time and/or if a fixed location policy is used to assign a storage space to a product. (Yao, Hay, et al., 2010) presents a mixed-integer nonlinear programming model and a heuristic for a joint facility location-allocation and inventory problem that incorporates multiple sources of warehouses intending to minimize the total cost. Here, a multi-product environment is considered where demand is stochastic and safety stock level is maintained. The solution gives the number and location of warehouses, allocation of products to fulfill demand, and the inventory levels of the warehouses. Several other metaheuristic algorithms such as genetic algorithm (Maglić et al., 2020), ant colony optimization algorithm, tabu search (Yao, Hay, et al., 2010), and simulated annealing (Muppani (Muppant) & Adil, 2008)are identified to be employed concerning the problem of warehouse and inventory management optimization. To maintain the production process smoothly, there is a need to deliver required materials from the warehouse to the production floor on time. Therefore, warehouse managers should allocate materials effectively by utilizing warehouse spaces optimally. Consequently, (Poon et al., 2011) have introduced a Production Material Demand Order Management System (PMDOMS) to help allocate resources of warehouses such as forklifts to fulfill material requirements of small-batch production. At the same time, PMDOMS can be used to prepare optimum warehouse routine & distribution plans such as plans for optimizing warehouse resources while minimizing the pick-up and delivery routes. For this model, automated data capturing technology and an advanced problem-solving technique are utilized by PMDOMS. In the end, this model will lead to an increase in the operational efficiency of the production floor. (Chuang et al., 2014) has proposed a class-based assignment policy aimed at improving order picking efficiency over randomized storage. At the same time, this model is based on data mining techniques. Through web statistics & rules, mining investigates the degree to which product categories exist. This model includes zoning of the storage area of warehouse storage space. Operation of this process happens by converting requested orders into Pallet-Case-Broken case (PCB) data and with batch picking. Further, (Poon et al., 2011) have found that zoning and class-based assignment policies tend to decrease travel distances by 24% while enhancing picking times by 21% compared to random assignment policies. Previous scholars have revealed several heuristic algorithms, but most scholars have presented work on picker to parts warehousing systems or automated storage and retrieval systems. Among such studies, (J. C. H. Pan, Shih, Wu, et al., 2015) has developed a pick and pass system with multiple pickers to identify relevant space for each product item and balance the workload of picking zones of the warehouse. This is based on a heuristic method. Through the presented model, it is possible to determine the optimum storage space of the warehouse to mitigate shortages that happen when the warehouse is to be picked.

The concept of batching policy will allow the measurement of how orders requested from the warehouse are consolidated to form batches. Many previous studies have highly focused on order batching policy at manual warehouses and also on its impact on the pick and pass system. (Wutthisirisart et al., 2015) has proposed a group genetic algorithm that is based on the order batching approach. It will lead to a balance in the workload of each picking zone and minimize the number of batches in a pick and pass system. Finally, it will optimize the space of every zone & manage to increase the overall performance of the warehouse processes.

Most of the cases found in the literature are based on layout methods for traditional warehouse optimizations. It is identified within this review that, most of the models and case studies previously established are challenged by the present dynamic era of warehouse operations. (Horta et al., 2016) has identified a necessity for a new approach to optimizing the internal layout of the warehouse. Consequently, has developed a model regarding the cross-docking basis. It is a mathematical programming approach that uses a min-max formulation which provides an efficient internal layout at a cross-docking warehouse. The output of the model is a unique internal layout that optimizes the warehouse space, focusing on multi-picking characters in the case of storage assignment in the warehouse. (Kovács, 2011) has proposed a class-based storage policy for minimizing the cycle time of the requested order and average effort of picking or a combination of those factors. The finding of this model is, it will achieve a trade-off between the two criteria mentioned and it will help achieve the lowest cycle time with an average effort of picking.

(Hwang, 2006) has especially considered a situation with a certain performance evaluation model for order picking policy when designing warehouses by minimizing the travel distance to and from warehouses. The proposed mathematical & simulation-based model considers the probabilistic demand of the organization and the frequency of picking. It is identified that it considers important factors such as the size of a particular warehouse, rack size, and the number of routines per day. The conclusion of this model is achieving an optimal design of the warehouse. Therefore, this achieves optimum space utilization as well.

(Elbert et al., 2017) has mainly focused on the case of human interaction in the manual order picking systems because human movement impact makes deviations from pre-scheduled routines at developed routines policies. The paper presents an agent simulation model to quantify the mentioned deviations due to human interactions. The presented model compares several routines with extensive simulation experiments. Finally, it identifies the relevant deviations and provides important insights for warehouse managers to optimize the routines. At the same time, indirectly optimizes the space utilization of the warehouse. Traditional warehouse management systems, classify similar orders as the same batch to fulfill the requested order. Through that, it is expected to achieve optimized picking routines. (J. C. Pan et al., 2015) have introduced a method to improve the particle swarm optimization algorithm to solve the joint order batching and picker Manhattan routing problem faced in a dynamic business environment. Further, the work parallelly decides the order batching allocation and the routine distance of the picker. This is identified as a model applicable to more complex warehouses. It can increase the efficiency of warehouse routines. (Gue et al., 2014) has developed a warehouse-related model based on networks. This model highly considers individual pallet locations, their movements, interactions with specified cross aisles to investigate the expected routine distance of an established warehouse design. This model is more applicable to get estimates of the appropriate angles of cross aisles and picking aisles for multiple and deposit points in the warehouse which consists of unit loading facilities. The output of this type of model will allow the reduction of the expected routine distances by using alternative designs. Consequently, it is subjected to decline as P & D points while increasing the number of dispersions. Overall, the model will lead to optimization of the storage space.

One set of warehouse-related issues is based on multi-level storage location assignment problems for SKU pallets by considering divisible locations in the first level to enhance the picking operations process and to reduce the travel time related to the routines. For this purpose, a mathematical model is presented in (Operational, 2018) to minimize the travel distance while maximizing the space utilization of the warehouse. At the same time, it mitigates deterioration due to the stack priority. (Perera et al., 2021)proposes a linear programming model to optimize the usage of warehouse storage space by efficient palletizing. The quantity of total pallets required per day is derived based on the available demand per day and other constraints related to warehousing operations such as manufacturing output, warehouse capacity, product category, etc. in a multi-product manufacturing context. The model could be simply implemented using Microsoft Excel add-in, Solver, and the Solver Engine - Simplex Linear Programming.

In a warehouse scenario, put away and picking operations play an important role. It will lead to the determination of the cost of operating & response time of a customer, at the same time, it impacts space usage of the warehouse. Therefore, (Öztürkoğlu, 2020) has focused on put-aware operations in the pallet in & pallet out at the block-stacking warehouse. The paper presents a unique bi-objective mathematical model and a constructive heuristic algorithm to allocate items into the storage lanes, intending to maximize warehouse storage space and minimize the total travel distance for routines.

The warehouse can be considered the hub and main crucial point where control and handle all types of raw materials, work in progress inventories, and finishing goods. Based on the available facilities of a particular warehouse, such as palletizing methods, the layout of the warehouse space optimizations of the warehouse could be unique. After that, considering available optimization techniques and the scope, more critical to pay attention to optimizing the use of warehouse storage space using the palletization function, especially in a situation where the demand is previously known by the warehouse managers in the form of orders. Most of the previous scholars have explored that proper solutions and optimization techniques using heuristics and several algorithms. Subsequently, the methods or models that directly engage with the operation research area have not been explored so far to achieve the optimum warehouse storage space. Not only that but also how those operations relate models or techniques directly reduce the unnecessary utilization of warehouse spaces has not been explored. Therefore, the main research objective of this study is to propose an effective linear programming model and goal programming model to use warehouse storage space by efficient palletizing optimally.

Table 1: Summary of Models

Reference	MM	GA	Used Mathematical Methods	Statistical Techniques	Heuristic Algorithm (HA)
(Poon et al., 2011)	Y	Y	TSP		
(Chuang et al., 2014)	Y			Data mining technique	
(J. C. H. Pan, Shih, Wu, et al., 2015)	Y	Y	NLP, B&B, MIP	Simulation model (on FlexSim)	dynamic heuristic procedures
(J. C. H. Pan, Shih, & Wu, 2015)	Y	Y (GGA)	Jackson network model, GI/G/m queuing network model, DP	Simulation model (on FlexSim)-experimental design and a simulation model	S-shape heuristic method
(Horta et al., 2016)	Y		IP	Cluster Analysis	
(Che et al., 2017)	Y		bi-objective MINLP, QAP		
(Öztürkoğlu, 2020)	Y		bi-objective mathematical model		Heuristic
(Kovács, 2011)	Y		MIP	Correlation between the request probabilities of items in the warehouse	cluster-first/zone second heuristic
(Hwang & Cho, 2006)	Y			Simulation models, Normal distribution and Binomial distribution	
(Elbert et al., 2017)				Simulation model	heuristic routing policies
(Lin et al., 2016)		Y	PSO, ACO, order picking and picker Manhattan routing problem, ImPOS		

(Gue & Meller, 2014)	Y		PSO, Neighborhood Topologies, Star and Ring Topologies, NLP, non-differentiable multi-model problems,		
(Battini et al., 2016)	Y		bi-objective approach, Pareto frontiers, non-dominated optimization	Pareto frontier for alternative values of the cumulative popularity function, Simulation & uniform distribution random variables	
(Ballesteros-Riveros et al., 2019)	Y	Y	ACO, B&B		heuristic based on a B&B
(Manzini et al., 2011)	Y	Y	what-if multi-scenario analysis	systematic multi-step approach, simulation of vehicles’ routes, parametric class-based frequency analysis and/or a non-parametric continuous frequency-based analysis, clustering rule	
(Geraldes et al., 2012)	Y		(MINLP), multi-stage hierarchical decision approach	simulation technique	HA-jointly determines the size of the functional areas and the allocation
(Karasek, 2013)			multiple-block warehouse, TSP, ACO, Annealing-algorithm	Analytical and simulation models	Composite heuristic, TSP heuristics, Markov Decision Process-based heuristics
(Berg & Zijm, 1999)	Y		Lagrangian multiplier technique, polynomial-time DP	Independent stochastic method, sophisticated sorting techniques	heuristic into a framework for determining the size of the forward area

					Wanigasuriya and Hettiarachchi (2022)
					together with the allocated products
(Sanei et al., 2011)	Y		ILP, B&B, PSO		HA-based on the B&B, heuristic, and metaheuristic algorithm
(Guerriero et al., 2013)	Y		LP, assignment problem, PSO		the heuristic is defined to solve large scale scenarios in a reasonable amount of time
(Heragu et al., 2005)	Y		MILP, binary integer variables, B&B		HA that jointly determines product allocation to the functional areas in the warehouse as well as the size of each area using data readily available to a warehouse
(Caserta et al., 2011)	Y		DP		hybrid metaheuristic, linking together mathematical programming techniques with heuristic schemes
(Maglić et al., 2020)	Y	Y	discrete optimization, ILP	Simulation model	heuristic procedures and binary programming
(Pohl et al., 2009b)			one-way travel expression and NLP		
(Yao, Lee, et al., 2010)	Y		MINLP		iterative heuristic (Utilizing approximation and transformation techniques)

Source: Compiled by the author

Table 2: Abbreviation table

Abb	Methos	Abb	Method	Abb	Methos
MM	Mathematical Model	NLP	Nonlinear Programming	ACO	Ant Colony Optimization
GA	Genetic Algorithm	MINLP	Mixed Integer Nonlinear Programming	B&B	Branch and Bound
GGA	Group Genetic Algorithm	ILP	Integer Linear Programming	PSO	Particle Swarm Optimization
TSP	Traveling Salesman Problem	DP	Dynamic Programming	MIP	Mixed Integer Programming
ImPOS	Improved PSO	QAP	Quadratic Assignment Problem	HA	Heuristic Algorithm

Source: Compiled by the author

METHODS

The model proposed concerns the optimal use of the warehouse storage area only in ground level (without stack storage) with relevance to the palletization function in its supply chain. The quantity of total pallets required per day is derived based on the available pre-demand per day, to best utilize warehouse space. Using the models proposed both Linear programming and Goal programming, the number of pallets required to be stored in the warehouse could be found in advanced, which allows optimal usage of warehouse storage area and satisfies all the demand in a multi-product manufacturing context.

Consider the sample situation represented in table.1, where the number of pallets that can fulfill a certain amount of demand is shown. Assume that each type of pallet requires an equal storage area and can only be stacked at ground level (no stack is placed over another stack). According to the table below (Table 1:), the highest number of pallets (22 pallets) is required for demand for 1000 units and the other three types of demand quantities require an equal number of pallets (12 pallets).

Table 3: Number of Pallets for certain order

Demand	Pallet Types			
	50	45	35	25(Manual)
1000	2	20	-	-
600	12		-	-
580	4	8	-	-
400	-	-	10	2

Source: Compiled by Author

For a sample case taken from the cement manufacturing industry, where the maximum capacity of the warehouse is 30000 units and three product types (AA, AB, AC) are available with 10000 units' capacity from each product type, and daily demand requirements are as below (Table 2).

Table 4: Daily demand requirement

Product	Order Type			
	1000	600	560	400
AA	10	5	3	0
AB	8	5	4	2
AC	4	3	1	5

Compiled by the author

METHOD-I: Optimize space of palletizer warehouse using a Linear Programming (LP) model

Decision variable:
x_{ij}= the number of k[th] pallet type from i[th] product type (for i=1, 2,.,n & k=1,2,….,r)
Objective function:
The maximum number of pallets is needed to fulfill the demand from each product type.
$Max\ Z = C_k \sum_{i=1}^{n} x_{ik}$ (for k=1,2,….,r) -------------------------------------- (1)
Constraints:
The produced total number of pallets from each product type must be equal to or less than the daily production availability of each type.
$\sum_{i=1}^{n} C_k x_{ik} \leq P_i$ (for k = 1,2,…,r)---(2)
The total number of produced pallets from each product type must be equal to or less than the capacity of the warehouse.
$\sum_{i=1}^{n} x_{ik} \leq \sum_{i=1}^{n} A_i Y_{ij}$(for k=1,2,….,r & j=1,2,….,m) -----------------------(3)
The number of pallets according to the placed order from each type must be equal to or less than the capacity of the warehouse.
$C_k \sum_{i=1}^{n} x_{ik} \leq M$ (for k=1,2,….,r)--(4)
The number of pallets is an integer and greater than zero. (Non-negativity constraint)
$x_{ik} \geq 0$ (for i=1,2,…,n & k=1,2,…,r)--(5)

Main Parameters:
n = total number of brand types (product types)
m = total number of order types
r = total number of pallets groups
C_k = the quantity type of pallet in group k (for k=1,2,…,r)
P_i = the available capacity of product type i (for i=1,2,…,n)
Y_{ij} = the number of j[th] order type from i[th] product type according to daily requirement (for i=1,2,…,n & j=1,2,…,m)
Objective function

$$Max\ Z = 50\sum_{i=1}^{3} x_{i1} + 45\sum_{i=1}^{3} x_{i2} + 35\sum_{i=1}^{3} x_{i3} + (0.2 * 25\sum_{i=1}^{3} x_{i3})$$

Constraints
For product type AA

$$50x_{11} + 45x_{12} + 35x_{13} + 0.2 * 25x_{13} \leq 10{,}000$$

For product type AB

$$50x_{21} + 45x_{22} + 35x_{23} + 0.2 * 25x_{23} \leq 10{,}000$$

For product type AC

$$50x_{31} + 45x_{32} + 35x_{33} + 0.2 * 25x_{33} \leq 10{,}000$$

For pallet type I

$$x_{11} + x_{21} + x_{31} \leq 2(Y_{11} + Y_{21} + Y_{31}) + 12(Y_{12} + Y_{22} + Y_{32}) + 4(Y_{13} + Y_{23} + Y_{33})$$

For pallet type II

$$x_{12} + x_{22} + x_{32} \leq 8(Y_{11} + Y_{21} + Y_{31}) + 20(Y_{13} + Y_{23} + Y_{33})$$

For pallet type III

$$x_{13} + x_{23} + x_{33} \leq 10(Y_{41} + Y_{42} + Y_{43}) +$$

Lower bound for Decision variables

If 600 units order placed then lower bound for pallet type I where i=1,1,3

$$x_{i1} \geq 12$$

If 560 units order placed lower bound for pallet type I where i=1,2,3

$$x_{i1} \geq 4$$

If 1000 units order placed, then lower bound for pallet type I where i=1,2,3

$$x_{i1} \geq 2$$

If 1000 units order placed, then lower bound for pallet type II where i=1,2,3

$$x_{i2} \geq 20$$

If 560 units order placed, then lower bound for pallet type II where i=1,2,3

$$x_{i2} \geq 8$$

If 400 units order placed, then lower bound for pallet type III where i=1,2,3

$$x_{i3} \geq 10$$

Integer constraint for decision variables i=1,2,3 and k=1,2,3,4

$$x_{ik} = integer$$

Non-negativity constraint for daily requirement and production where i=1,2,3 and j=1,2,3,4

$$Y_{ij} \geq 0 \ \& \ P_i \geq 0$$

METHOD-II: Optimize space of palletizer warehouse using a Goal Programming (GP) model

Decision variable:

x_{ik} = the number of kth pallet type from ith product type (for i=1,2,….n & k=1,2,….,r)

Auxiliary variable:
Di = D_i^+ - D_i^- , where D_i^+ & D_i^->=0 (extra variables that helpful for formulating the model)
Objective function: The minimize total penalty values.
Min Z = $\sum_{i=0}^{n}(D_i^+ - D_i^-)$ (for i=1, 2, …., n) ------------------------------(6)
Goals:
The maximum number of pallets is needed to fulfill the demand from each product type.
$C_k \sum_{i=0}^{n} X_{ik} - (D_g^+ - D_g^-) = M$ (for k=1, 2, …., r & g =1,2,…d)-------(7)
The produced total number of pallets from each product type must be equal to or less than the daily production availability of each product type.
$\sum_{i=1}^{n} C_k X_{ik} - (D_g^+ - D_g^-) = P_i$ (for k=1,2,….,r & g=1,2,…d)------------(8)
Constraints:
The total number of produced pallets from each product type must be equal to or less than the daily pallets requirement from each product type.
$\sum_{i=1}^{n} X_{ik} = \sum_{i=1}^{n} A_i\, Y_{ij}$ (for k=1, 2, ……, r & j=1, 2, …., m) -------------(9)
The number of pallets according to the placed order from each product type must be equal to or less than the capacity of the warehouse.
$C_k \sum_{i=1}^{n} X_{ik} \leq M$ (for k= 1, 2, …., r) ---(10)
The number of pallets is an integer and greater than zero.
$X_{ik} \geq 0$ (for i=1, 2, …, n & k=1, 2, …., r) ---------------------------------(11)

Main Parameters:
n = total number of brand types (product type)
m = total number of order types
r = total number of pallet groups
d= total number of goals
D_g = auxiliary variable for g[th] goal (for g = 1,2,…….d)
C_k = the quantity type of pallet in group k (for k = 1, 2, …, r)
P_i = the available capacity of product type i (for i=1, 2, …., n)
Y_{ij} = the number of j[th] order type from i[th] product type according to daily requirement (for i=1,2 … ,n & j=1,2,….,m)
M = the capacity of warehouse

Objective Function:
Min $Z = D_1^+ + D_2^+ + D_3^+ + D_4^+$
Goal 1: Optimize warehouse space
$50\sum_{i=1}^{3} X_{i1} + 45\sum_{i=1}^{3} X_{i2} + 35\sum_{i=1}^{3} X_{i3} + (0.2*25\sum_{i=1}^{3} X_{i3}) - (D_1^+ - D_1^-) = 30000$
Goal 2: For product type AA
$50X_{11} + 45X_{12} + 35X_{13} + 0.2*25X_{13} – (D_2^+ - D_2^-) = 10000$
Goal 3: For product type AB
$50X_{21} + 45X_{22} + 35X_{23} + 0.2*25X_{23} – (D_3^+ - D_3^-) = 10000$

Goal 4: For product type AC
$50X_{31} + 45X_{32} + 35X_{33} + 0.2*25X_{33} – (D_4^+ - D_4^-) = 10000$

Constraints:
For pallet type I

$$x_{11} + x_{21} + x_{31} \leq 2(Y_{11} + Y_{21} + Y_{31}) + 12(Y_{12} + Y_{22} + Y_{32}) + 4(Y_{13} + Y_{23} + Y_{33})$$

For pallet type II

$$x_{12} + x_{22} + x_{32} \leq 8(Y_{11} + Y_{21} + Y_{31}) + 20(Y_{13} + Y_{23} + Y_{33})$$

For pallet type III

$$x_{13} + x_{23} + x_{33} \leq 10(Y_{41} + Y_{42} + Y_{43})$$

Lower bound for decision variable:
If 600 units order placed lower bound for pallet type I where i=1, 2, 3

$$x_{i1} \geq 12$$

If 560 units order placed lower bound for pallet type I where i=1, 2, 3

$$x_{i1} \geq 4$$

If 1000 units order placed lower bound for pallet type I where i=1, 2, 3

$$x_{i1} \geq 2$$

If 1000 units order placed lower bound for pallet type II where i=1, 2,3

$$x_{i2} \geq 20$$

If 560 units order placed lower bound for pallet type II where i=1, 2,3

$$x_{i2} \geq 8$$

If 400 units order placed lower bound for pallet type III where i=1, 2,3

$$x_{i3} \geq 10$$

Integer constraint for decision variables

$$x_{ik} = Integer$$

Non-negativity constraints

$$Y_{ij} \geq 0 \;\&\; P_i \geq 0$$

RESULTS

METHOD-I: Optimize space of palletizer warehouse using a Linear Programming (LP) model
For the sample scenario considered, the solution is feasible and is an integer solution within tolerance, with a maximum value for objective function (Max Z) as 29,965 units and optimal decision variables values shown in table 5. All constraints are satisfied. Optimal decision variables values are shown in table 5. All Constraints are satisfied.

Table 5: Optimal solution for decision variables using LP

Xik	Pallet Type (k)		
Product (i)	I(50)	II (45)	III (35)
1 (AA)	12	200	10
2 (AB)	68	137	10
3 (AC)	174	20	10

Source: Compiled by the author

According to the optimal values of decision variables it is found that, manufacture from each product type (product) AA = 10,000 units, AB = 9,965 and AC = 10,000 units should be produced. The software used is the Microsoft Excel add-in, Solver, using the Solver Engine - Simplex LP, where the Solution Time was 0.047 Seconds.

METHOD-II: Optimize space of palletizer warehouse using a Goal Programming (GP) model
For the sample scenario considered, the solution is feasible and is an integer solution within tolerance, with a maximum value (Goal 1 result) for storing capacity as 29,900 units and optimal decision variables values shown in table 6. All constraints are satisfied.

Table 6: Optimal solution for decision variables using GP

Xik	Pallet Type (k)		
Product (i)	I(50)	II (45)	III (35)
1 (AA)	12	209	0
2 (AB)	46	162	10
3 (AC)	174	20	10

Source: Compiled by the author

According to the optimal values of decision variables, it is found that manufacture from each product type (product) AA (Goal 2) = 10000 units, AB (Goal 3) = 9950 and AC (Goal 4) = 9950 units should be produced.

DISCUSSION

Through the literature review of this study, it was observed that modern warehouse operations significantly differ from traditional warehouse operations in terms of flexibility, integration, customer orientation, digitalization, data availability and information sharing, reduction or complete elimination of the need for warehouses with the direct involvement of consumers and suppliers, on the other hand – the rise of profitable global companies that operate from only a warehouse with large scale inventories, just in time production concepts, etc. Therefore, it is identified that most of the models and frameworks are subject to significant changes to apply to modern warehouse operations in a dynamic and modern scale and usage. One aspect that is highlighted is the need for integrated approaches for warehouse operational problems which are significantly correlated. It is apparent that warehouse operations are complex and interrelated and do not generate globally optimal results when considered or optimized in isolation. For instance, SKUs are retrieved from their storage positions based on customers' orders and moved to the accumulation and sorting area or directly to the shipment area. The picked units are then grouped by customers' order, packaged and stacked on the right unit load, and transferred to the shipping area. At each level, multiple decisions are interrelated and therefore it is necessary to cluster relevant problems that are to be solved simultaneously (M. T. Hompel & Schmidt, 2007). To derive considerable organizational value, it is needed to position the operations of a modern warehouse in the context of the complete supply chain.

As warehouse operations do not directly add value to an organization's activities or product, unless the activities of providing value-added services such as the fulfillment of individual customer orders, packaging of goods, after-sales services, repairs, testing, inspection, and assembly (Clarke & Wright, 1964) occur at warehouse level in a supply chain, the costs associated with it, in terms of capital, time and resources, pose a significant challenge to modern organizational profitability. This is specially attributed to the feature that warehouses are one of the key points of logistic processes where a substantial possibility of controlling and reducing logistics costs exists (Heragu et al., 2007). The design of a warehouse itself is a highly complex problem. It includes a large number

of interrelated decisions involving warehouse processes, warehouse resources, and the organization of the warehouse (Clarke & Wright, 1964) and shapes the efficiency and effectiveness of the entire supply chain once in operation.

It is also identified through this study that most of the warehouse operations possess a degree of uniqueness based on the industry of operation and organizational policies. In addition to factors such as the size, quantity of the products stored, and the handling characteristic of products or their product carriers (Ozgormus & Smith, 2020), picking or retrieval functions of warehouse operations; homogeneity or heterogeneity (Pohl et al., 2009b), etc. Therefore, it is seen that the scope of possible research areas is narrowed down to problem-specific solution findings. Several models based on heuristic and metaheuristic algorithms have delivered promising results in the area of warehouse space optimization, as identified through this study. A further summary of models is presented in Table I Summary of Models, where some recent, cited papers are compared on the algorithm used, statistical validity/ techniques used, and objective function(s) modeled.

CONCLUSION

Warehousing involves a series of interrelated material handling activities that take place within a warehouse in a supply chain, a majority of which are typically identified to be assignment problems. Optimization of these activities is essential through continuous evaluation of warehouse operational efficiency to ensure consistency in terms of fulfilling the market's demands and alignment with management strategies. The review presented in this paper presents a comparative analysis of optimization models used for warehouse space allocation proposed in recent literature. Through the analysis presented in this review, it was identified that considerable challenges exist in applying these models even at present due to the vast amount of information required to be processed, the significant number of possible alternatives, and the degree of integration of decisions required in the modern warehousing context. It is also identified that the subject follows a problem-specific research trend as warehousing policies and operations display a certain degree of uniqueness between organizations and business models.

This paper presents simple and effective LP and GP models to optimally use warehouse storage space by efficient palletizing. The quantity of total pallets required per day is derived based on the available demand per day and other constraints related to warehousing operations in a multi-product manufacturing context. Requiring moderate computational efforts and time, the model was able to generate optimal solutions for almost all examples analyzed. This suggests that the proposed LP and GP are effective solutions to the problem of optimizing the number of pallets needed to fulfill the demand from each product type, maximizing warehouse space utilization. The approach proposed in this paper can be used as a basis for further optimization of basic warehousing functions in a practical environment where the solution space is not exponentially large, such as that of a single warehouse.

LIMITATIONS AND FUTURE RESEARCH

The proposed models of this study focus on optimizing ground floor space utilization in a particular manufacturing organization, there is a need for future research to develop a model for optimizing not only ground floor but also storage space available for vertical stacking.

The proposed model of this study mainly takes daily demand as input. Using statistical methods (Ex. Forecasting), logistics and distribution managers could determine order requirements in advance. The research could be further extended to incorporate forecasted order quantity as inputs to the model. Furthermore, the proposed model of this study considers the fact that objects that are stacked on to a pallet are homogenous. The model could also be tested and improved to perform in a situation where the objects to be stacked on to the pallet are heterogeneous and are possible avenues for future research.

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